Exhibit 24

AD VALOREM AND UNIT TAXES COMPARED

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A series of articles in this *Journal* during 1939 and 1940 dealt with the comparative incidence of a unit and an ad valorem sales tax.[1] This discussion analyzed at length the comparative changes in price under a unit and an ad valorem tax which are, in some sense, matched in their relation to the initial (i.e., pre-tax) price. But little if any attention has been paid to what seems to us the more relevant comparison, namely that between unit and ad valorem levies which are equivalent in terms of final total tax yield. Also, the discussion has suffered from the use of a rather unwieldy diagrammatic technique.

In Section I of this note we demonstrate the relationship between the prices resulting from equal-yield unit and ad valorem taxes. In Section II we show how certain conclusions from the earlier articles may be extended by reference to total yield conditions.[2]

I

Under pure competition it is obvious that unit and ad valorem taxes which result in equal yields will also result in equal final prices. The comparison of changes in monopoly price was made by Wicksell nearly fifty years ago for the case of constant cost.[3] Wicksell argued that the increase in price would always be greater under the specific tax, which is therefore inferior to the ad valorem tax on welfare grounds. Unfortunately this discussion has remained almost entirely unnoticed. The following argument covers more or less the same ground, but is more general. For the monopoly case the following propositions apply:

1. The yield from any given unit tax is always smaller than the

1. See Donald W. Gilbert, "The Shifting of Sales Taxes," this *Journal*, February 1939, pp. 275–85; E. D. Fagan and R. W. Jastram, "Tax Shifting in the Short Run," this *Journal*, August 1939, pp. 562–89; and further discussion by B. H. Higgins, R. W. Jastram, John F. Due and W. D. Gilbert in this *Journal*, August 1940, pp. 665–93. See also John F. Due, *The Theory of Incidence of Sales Taxation* (New York: King's Crown Press, 1942), especially chap. V and the bibliography on pp. 245–48; and von Mering, *The Shifting and Incidence of Taxation* (Philadelphia: Blakiston Co., 1942), pp. 36–40.

2. Following the earlier discussion, this analysis is limited to the short-run adjustment and deals with the problem in a partial equilibrium setting only

3. Knut Wicksell, *Finanztheoretische Untersuchungen* (Jena, 1896), pp. 15–21.

yield from the ad valorem tax which would result in the same final output and price.

2. The maximum yield which may be obtained from a unit tax is smaller than the maximum yield possible from an ad valorem tax.

3. If the same yield is obtained from a unit and an ad valorem tax, the final price will be higher (the output smaller) under the unit tax.

Proof of Proposition 1. Suppose a tax of t dollars per unit has been imposed, with the result that the monopolist maximizes profits at an output x_t and price p_t. At this output marginal production cost equals marginal revenue net of tax:

$$MC(x_t) = MR(x_t) - t.$$

Let the unit tax t be replaced by an ad valorem tax rate of r (fraction of gross revenue) which tax results in the same final price and output.[4] It must follow that marginal revenue net of tax still equals marginal production cost at output x_t:

$$MC(x_t) = MR(x_t) - r\,MR(x_t).$$

Equating the right sides of the two equations and solving, we obtain

$$r = \frac{t}{MR(x_t)}.$$

The yield of the unit tax is given as

$$Y_t = tx_t,$$

while the yield of the ad valorem tax is

$$Y_r = r\,p_t x_t = \frac{t}{MR(x_t)}\,x_t p_t.$$

The difference between the ad valorem yield and the yield of the unit tax will then be

$$Y_r - Y_t = tx_t\left(\frac{p_t}{MR(x_t)} - 1\right).$$

Since the yield of the specific tax (tx_t) is positive and since price exceeds marginal revenue, the right side of the last expression is positive and the yield of the ad valorem tax exceeds that of the unit tax.[5]

4. Throughout our discussion the ad valorem tax rate r is defined as a per cent of gross revenue. However, many tax laws are written so as to specify a rate which applies to gross revenue net of tax. Our propositions, of course, hold in either case and the proofs are similar. If s is such a tax rate, we simply substitute r for s, where $r = \frac{s}{1+s}$.

5. To be formally correct, it should be added, of course, that Proposition 1 holds only for tax yields above zero.

We may further note that the proof of Proposition 1 also serves to prove the competitive case. If price always equals marginal revenue, the right side of the last expression is identically zero, and the yields are always equal.

Although Proposition 1 is completely general, it may be illustrated for the linear case as shown in Figure I. Let OS be the marginal cost schedule, and FD and FA be the demand and marginal

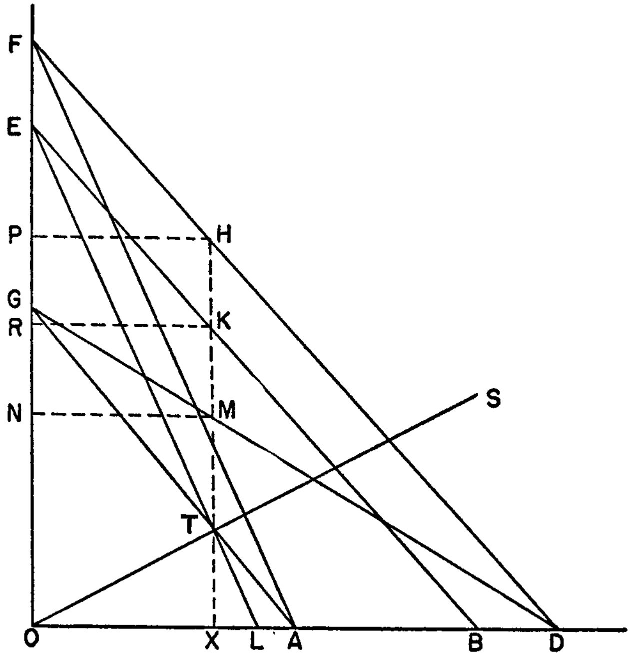

FIGURE I

revenue schedules before tax. After imposition of a unit tax equal to KH, the net demand and marginal revenue schedules become EB and EL respectively. The equilibrium output is OX and the tax yield, Y_t, is the area $RKHP$. For an ad valorem tax to result in the same price and output, the marginal revenue schedule net of ad valorem tax, AG, must pass through point T. The corresponding net average revenue schedule becomes GD. Average revenue net of tax is now XM ($= ON$) and tax yield is the area $NMHP$, which includes the area $RKHP$.

Proof of Propositions 2 and 3. Proposition 2 follows obviously from Proposition 1. Since for every unit tax the ad valorem tax leading to the same price has a higher yield, this must be true also for that unit tax which gains the maximum possible unit tax yield.

To demonstrate Proposition 3 we need merely recall that, up to maximum tax yield, the higher the yield is to be, the higher the final price will be. If an ad valorem tax results in a given yield, the

unit tax which would result in the same price and output gives a lesser yield. To obtain the same yield with a unit tax, the resulting price must therefore be higher.[6]

It follows from this that the same welfare argument which shows a sales tax (either unit or ad valorem) to be inferior to an equal-yield income tax, also shows the unit sales tax to be inferior to the ad valorem tax.[7]

II

We now turn to the comparison of prices which result from ad valorem and unit taxes which are matched at the initial price. As noted before, this is the framework within which most of the earlier discussion proceeded. The question here is as follows: Given a unit tax rate t and an ad valorem tax rate r, equal to $\frac{t}{p_0}$, where p_0 is the initial (pre-tax) price, in which case will the final equilibrium price be higher?

For the sake of ease in presentation, let us call such a pair of tax rates a "matched" pair of rates.[8] Let us denote the final equi-

6. This comparison, of course, holds only for ad valorem yields which do not exceed the maximum possible unit yield, and under the natural assumption that tax rates will not be pushed beyond the maximum yield point.

7. Certain qualifications apply equally in both cases. See I. M. D. Little, "Direct *vs.* Indirect Taxes," *Economic Journal*, Sept. 1951, and Milton Friedman, "The Welfare Effects of an Income Tax and an Excise Tax," *Journal of Political Economy*, Feb. 1952.

8. In Section I we have compared tax levies which would give rise to equal final yields. In this section we compare levies which would give the same yield at the initial price. Thus a unit tax t is matched with a gross ad valorem rate of $r = \frac{t}{p_0}$, or with a net rate of $s = \frac{t}{p_0 - t}$.

Due, *op. cit.*, chap. V, defines two distinct problems. A: the comparison of "two levies which impose the same burden at the old level of output" (p. 89); B: comparison between "a specific tax and an ad valorem tax the rate of which is the ratio of the specific tax to the old price" (p. 90). If the ad valorem rate referred to were a gross rate the two cases would, of course, be identical. Since, however, Due adopts the net rate for his analysis, the two problems differ. In problem A, t is compared with s where $s = \frac{t}{p_0 - t}$. In B, t is compared with s (again a *net* rate) where $s = \frac{t}{p_0}$! (Given a unit tax of 10 cents on a commodity selling for \$1, the liability of the matched ad valorem rate under A equals $\frac{1}{10} \times 100 = 10$ for the gross, and $\frac{1}{9} \times 90 = 10$ for the net definition. But under B the liability for the net ad valorem rate would be $\frac{1}{10}(100 - l) = \frac{100}{9}$).

In terms of welfare comparisons or optimal fiscal planning we believe the

librium prices achieved under the two taxes as p_t and p_r respectively. Symbolically the problem becomes: Given a matched pair of rates r and t, what can be said about the relative magnitudes p_t and p_r?

For the case of pure competition the matter is again quite simple. Final price must differ from marginal production cost by the amount of tax paid per unit. This amount will always be greater for the ad valorem tax which increases (per unit) as price rises. Thus for any matched pair of rates the increase in price will always be greater under the ad valorem tax.

For the case of monopoly the following propositions can be demonstrated:[9]

4. For any initial pre-tax price, there are always matched pairs of rates, such that p_t exceeds p_r. This is independent of demand and cost conditions.

5. There are matched pairs of rates such that p_r equals or exceeds p_t only if there is some point on the demand schedule where the corresponding value of marginal revenue equals or exceeds p_0.

6. If there is a matched pair which has the special property that $p_t = p_r$, then for any matched pair of higher rates, p_r exceeds p_t and for any matched pair of lower rates, p_t exceeds p_r.

7. Assuming linear marginal cost and average revenue schedules, and under the presumption that tax rates are not pushed beyond maximum yield levels, for any matched pair of rates, p_t will always exceed p_r.[1]

Proof of Proposition 4. For any unit tax t, marginal net revenue at the resulting price p_t equals marginal production cost:

$$MR(p_t) - t = MC(p_t). \qquad (1)$$

If now we replace the unit tax with an ad valorem tax $r = \dfrac{t}{p_0}$,

proper comparison should be between levies which give the same yield in final result. However, it seems to us that our (the usual) comparisons of Section II (Due's case A) are not irrelevant. Legislators may decide to levy a tax of x cents on a given article and then debate whether to use the unit or ad valorem tax. But we cannot conceive of a situation in which Due's case B would be of interest.

Due's B formulation appears to have been thought a necessary counterpart of the "net" legal definition of the ad valorem rate. As we have indicated, there is no such connection.

9. The essential elements of Propositions 4–6 were demonstrated by Fagan and Jastram (*op. cit.*, p. 584, see especially notes 7 and 8) by diagrammatic analysis.

1. Note that Proposition 7 is independent of whether marginal cost is increasing, constant, or decreasing and depends only on its linearity. As a matter of fact, the proposition is valid for any marginal cost function whose slope does not decrease. Geometrically, that is, Proposition 7 holds for any marginal cost curve that is linear or convex to the x axis.

where p_0 is the initial price, then p_t is no longer necessarily an equilibrium price. We have then

$$MR(p_t)\,(1 - r) = MC(p_t) + k, \tag{2}$$

where k is the difference between marginal net revenue and marginal production cost under the ad valorem rate. Substituting the value of $r = \frac{t}{p_0}$ and subtracting (1) from (2) gives

$$t\left(1 - \frac{MR(p_t)}{p_0}\right) = k. \tag{3}$$

Since initial marginal revenue $MR(p_0)$ is less than p_0, it is always possible to select a value of t sufficiently small to keep $MR(p_t)$ smaller than p_0, so that k is positive. In this case, the marginal net revenue obtainable at p_t under the ad valorem rate will exceed the marginal cost of production at p_t. Hence output will be expanded and the ad valorem equilibrium price p_r will be lower than p_t.[2]

Proof of Propositions 5, 6, and 7. Propositions 5 and 6 follow directly from 4. To prove 5 we note that k can be zero or negative only if there is some price at which $MR(p)$ equals or exceeds p_0. If there is such a price, a unit tax can be imposed which will give this price as an equilibrium, and the previous argument now shows that p_r would equal or exceed p_t.

To demonstrate Proposition 6 we need only note that k will be negative or positive as t, and hence p_t exceeds or falls short of some value at which k equals zero.

The proof of Proposition 7 follows from the readily demonstrated fact that with linear demand and cost schedules, the p_t corresponding to the unit rate which maximizes total yield will also be that price at which marginal revenue is equal to p_0. By Proposition 6, for any t less than this, p_t exceeds p_r.

The conclusion reached by previous writers has been that p_r may fall short of or exceed p_t, depending on the size of the levies and on cost and demand conditions. This is quite compatible with our Propositions 4, 5, and 6. But for the linear case, our Proposition 7 goes farther. By adding the perfectly reasonable assumption that rates are not raised beyond the level of maximum yield, we conclude that for any matched pair of rates the final price under the unit tax will *always* be higher.

Inasmuch as linear functions provide useful approximations to

2. The competitive case is a special case of the proof of Proposition 4. Under competitive conditions marginal revenue at p_t always *equals* p_t, and (for t above zero) must exceed p_0. Thus k is always negative and the ad valorem tax results in a higher price.

reality, it follows that we may expect the price increase always to be greater under the unit tax, the more since sales tax rates, with some possible exceptions, are well below maximum yield.

The conclusion under conditions of monopoly is thus precisely the opposite of that under conditions of pure competition. But this does not imply that the welfare preference between the two taxes depends on the state of competition. On the contrary, such preference must be based entirely on the equal yield comparison of Section I: the choice between the two taxes is a matter of indifference under pure competition and the ad valorem tax is preferable under monopoly.

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